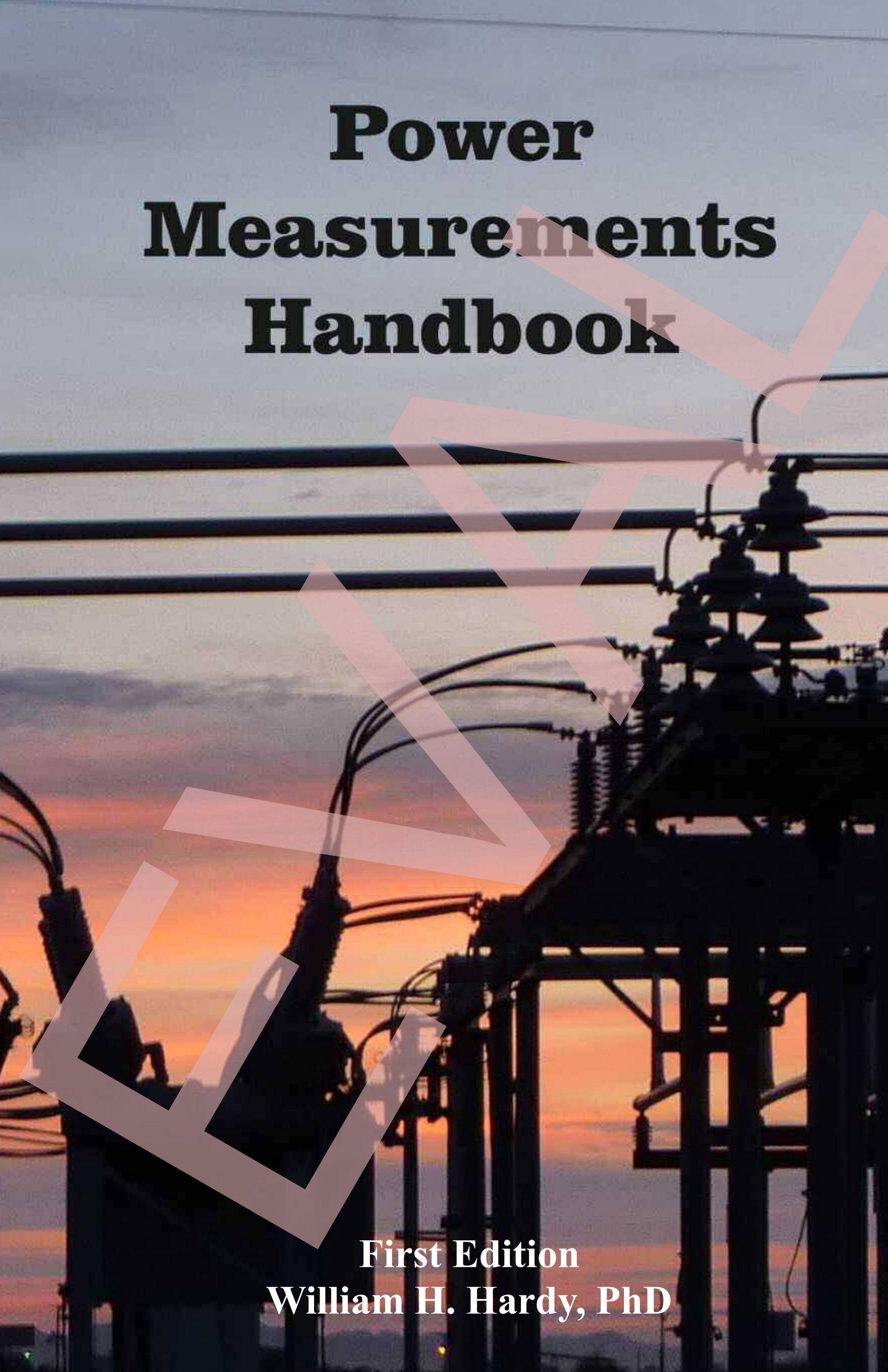


Power Measurements Handbook

First Edition

William H. Hardy, PhD

About the Author

The ***Power Measurements Handbook*** and www.PowerMeasurements.com are a “retirement” project of William H. (Bill) Hardy. Bill is a scientist/entrepreneur who came to the power measurement industry a decade ago after a 30 years career in a variety of instrumentation industries. His focus has been bringing new technologies and capabilities to metering. He is a member of ANSI C12 Main and heads several subcommittees and working groups. Bill continues to teach at a number of meter schools across the country. The Power Measurements website has a library of technical reference materials created by Bill that are available free of charge.

Visit

www.PowerMeasurements.com

You’ll like what you find there!

Power Measurements Handbook

First Edition

Foreword

Welcome to the ***Power Measurements Handbook.*** For the last decade I've been working in the instrumentation sector of the electricity metering industry. When I entered the industry I looked for a reference book for all of those facts that you need on a daily basis. I didn't really find one. Over the years of work, teaching at meter schools and participating on various ANSI committees I accumulated a ton of information. Now I've taken all of those notes and presentations and put them together into a pocket reference.

I hope you will find it the **go to resource** that finds an honored place in your tool box, glove compartment or desk drawer.

William H Hardy

William H. Hardy

Feedback

I welcome comments or suggestions for improving this handbook. It is the first edition so it is very likely there may be some mistakes and omissions. Your feedback would be greatly appreciated.

Contact

Power Measurements
William H. Hardy
6386 Avington Place
Gainesville, VA 20155
Email: Bill.Hardy@PowerMeasurements.com

First edition

Safety Warning

Energized electrical circuits present serious safety hazards. All circuits in electricity meters, meter sockets, CT circuits, PT circuits and distribution lines should be considered **live hazardous** unless verified otherwise. All work on live hazardous circuits should be done only by trained personnel following appropriate safety procedures.

CT circuits pose special safety issues. Secondary CT circuits should never be opened if there is any primary current. Opening a CT secondary can result in very high voltage across the secondary terminals. This can lead to explosive arcs and other extremely hazardous conditions.

The tables, diagrams and other information in this handbook are intended to be a reference for the most common configurations encountered by metering personnel. Never assume that a circuit conforms to these diagrams unless it has been **fully** verified.

This handbook is not intended as a replacement for proper training and experience. Always be sure to follow all company safety guidelines.

Disclaimer

The author assumes no liability with respect to the use of any information included herein whether or not used in accordance with any instructions or recommendations of this handbook.

1st Printing: May 2013
2nd Printing: July 2013
3rd Printing: September 2013
4rd Printing: December 2013 (minor revisions and corrections)

A special note of appreciation to Brad Johnson and his team at Oncor who did an extremely thorough proof reading job of the 3rd Printing enabling the many corrections in this 4th Printing.

User's Guide

This Handbook is intended to serve as a reference for the metering professional. Hopefully it will be useful to a wide range of people—apprentice to senior engineer. I've tried to make it interesting as well as informative.

For even more up to date information on electricity metering and all topics related to power measurements, visit www.PowerMeasurements.com. You'll find a wide variety of resources including:

- Complete listings and links to all of the North American meter schools
- Copies of presentations presented by Power Measurements
- Current status of various metering related standards
- A compilation of technical information on metering

If you have any material which you think would be useful to the metering community please send a copy to Bill.Hardy@PowerMeasurements.com so it can be shared with the whole community.

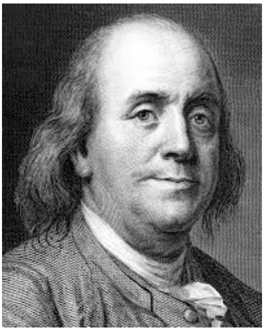
Table of Contents

A Little History

The recognition of static electricity goes back to the beginning of recorded history. The ancients recognized a strange property of amber and other substances. When rubbed with skins they attracted hair and small bits of debris.


Sir Thomas Browne

Ancient Egyptians (3000 BC) were aware that certain kinds of fish could create a “shock”. The scientific study of electricity really starts in the late 16th century when William Gilbert expands writings of Gerolamo Cardano and coins the Latin word *electricus* from the Greek word *elektron* (amber). The first use of the word electricity is ascribed to Sir Thomas Brown in 1646.


Benjamin Franklin

By the mid-1700s the Leyden jar had been developed as a storage device for static electricity and people had discovered that electric shock could be transmitted down a metal conductor. In 1752, Benjamin Franklin and William Watson discovered the idea of electrical potentials and established the convention of positive and negative. The 18th century comes to a close with the discovery of galvanism (after Luigi Galvani) who produced electrical current through the use of dissimilar metals (He created a very weak battery that could cause a frog’s leg to jump.).

Practical direct current electricity didn’t come until the early 19th century. In 1900 Alessandro Volta created the first high current battery. By 1809 large batteries were being built, electric ARC lights had been developed and new elemental metals were being discovered by electrically refining oxides. By mid-century much of the scientific theory of DC electricity and the connection between electricity and magnetism was established. Finally static and galvanic electricity were recognized as two aspects of the same phenomena.

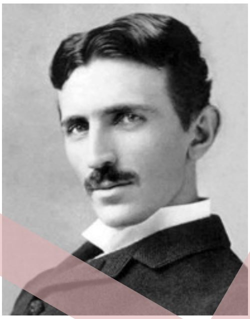

Michael Faraday


Joseph Henry

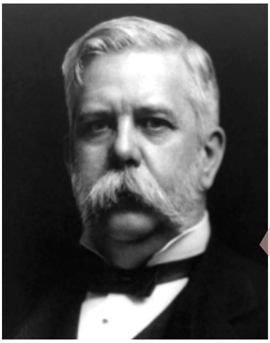
In the 1830s Michael Faraday and Joseph Henry discovered the concept of induction (transformers). By the 1850s people were doing electroplating, the telegraph had been invented, even the first facsimile machine capable of transmitting images was available.

The Electric Era

The idea of the electric dynamo (DC generator) was invented in 1860 and rapid improvements quickly made by Varley, Siemens and Wheatstone. At essentially the same time the reversed dynamo (electric motor) was created. During this same decade tremendous progress was made in the theory of electricity and magnetism by Maxwell, Hertz and Tesla. By the end of the 19th century we had most of the scientific understanding of electromagnetism.

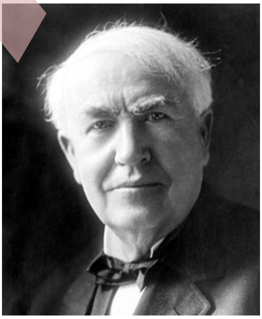


Nikola Tesla



George Westinghouse

By the 1870s the first commercial power plants had appeared in Paris. Arc furnaces were being used for metal production. Arc lights were in wide use. In 1879 Edison invented the electric light bulb. By 1880 he had established the Edison Electric Illuminating Company in NYC. Nikola Tesla who had worked for Edison, patented the AC induction motor and transformer in 1888. The patents for these two inventions, which are the foundation of the electric revolution, were sold to George Westinghouse. A great rivalry ensued between Edison and Westinghouse. Edison backed DC power distribution. Westinghouse backed AC power distribution. In 1886 Westinghouse deploys the first AC power system using step up and step down transformers in Great Barrington, MA. The same year Westinghouse wins the bid for the Buffalo City power system. 1887 sees development of three phase generators in the US and Europe. 1888 the first practical three phase motor is developed. In the 1890s three phase systems are installed in California. By the end of the century AC power systems are established as the standard world wide.



Thomas Edison

50 Hz or 60 Hz

In the early development of AC power every system (town) had its own unique frequency. These ranged from 16⅔ Hz to 133 Hz. Lighting systems worked best at higher frequencies. Small motors worked better at low frequencies and were difficult to fabricate above 60 Hz. Very large motors worked well at lower frequencies like 16⅔ Hz and 25 Hz. The first Niagara Falls plant generated 25 Hz. The early European plants operated at 40 Hz, 41⅔ Hz and 42 Hz. Over a relatively short period of time the dominant manufacturers established standards: 50 Hz in Europe and 60 Hz in the US (except for Southern California which stayed 50Hz until 1948.).

Static Electricity

For thousands of years people realized that something strange happened when you rubbed some materials with certain types of cloth. Rub a glass rod with silk and the silk would be attracted to the glass rod. Rub wax with wool and they would attract. Rub two pieces of glass with silk and the two pieces of glass would repel each other.

Ben Franklin proposed that the cloths were wiping some invisible fluid from the solids. Thereby creating an excess on the cloth and a deficit on the solid. The terms “negative” and “positive” come from this concept of removing fluid. Little did he know how much confusion would result from an almost correct hypothesis. Only much later did scientists recognize the existence of the real nature of the “fluid”. Subatomic particles called electrons, parts of atoms, compromise the fluid. They get transferred from the wool to the wax by rubbing, just the opposite of Franklin’s conjecture.

Facts: The unit for charge is the coulomb.
1 coulomb = 6.2415×10^{18} electrons

Electrons can move from one point to another. Their ease of movement is known as electric “conductivity”. Materials through which electrons move easily are called “conductors”. Materials through which electrons move with great difficulty are called “insulators”.

Facts: The unit of resistance is the ohm (Greek: omega Ω)
Conductivity generally represented by ρ (Greek: rho) has units of ohm•meters.
Resistivity is the inverse of conductivity. It is represented by σ (Greek: sigma). It has units of siemens per meter.

Conductance (shown in parenthesis below) is very large for good conductors and very small for good insulators.

Conductors include: silver($6.3x10^{7}$), copper($6.0x10^{7}$), gold($4.1x10^{7}$), aluminum($3.5x10^{7}$), iron($1.0x10^{7}$), steel($7.0x10^{6}$), sea water(4.8)

Insulators include: drinking water($5x10^{-2}$ to $5x10^{-4}$), pure water($5.5x10^{-6}$), glass(10^{-11} to 10^{-15}), rubber(10^{-14}), air($5x10^{-15}$), dry wood(10^{-14} to 10^{-16}), teflon(10^{-23} to 10^{-25})

The human body is a conductor, a very complicated one. Most of the resistance is skin resistance, while the body core has quite low resistance. If the skin is wet or damaged then the resistance from hand to hand or hand to foot is only a few hundred ohms. Under these conditions severe burns or fatal electrocution is highly likely at utility voltages.

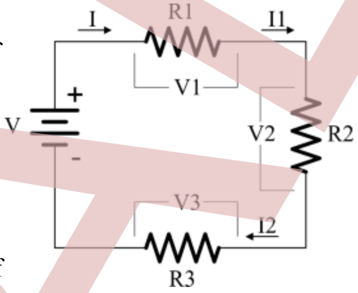
Direct Current Circuits

When there is a potential difference (voltage) between two points, a current (stream of electrons) will flow between the two points if they are connected by a conductor. Ohm's Law states:

$I = V/R$ Current = Voltage divided by Resistance

The higher the voltage or lower the resistance the more current flows.

Kirchoff's voltage law (KVL) states: The sum of all voltages around a circuit loop is zero.



Kirchoff's current law (KCL) states: The sum of all currents into a node of a circuit is zero.

$V = V1 + V2 + V3$

$I = I1 = I2 = I3$

$V = (I1/R1) + (I2/R2) + (I3/R3)$

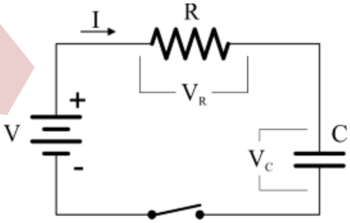
Joule's Law defines the relationship between voltage, current and power. In the simple case of a resistor, power is the heat dissipated.

$P = I^2R$ $P = E^2/R$ $P = VI$

There are two additional types of linear components that are of common interest: capacitors and inductors. A capacitor stores electric charge. The stored charge produces a voltage across the capacitor given by:

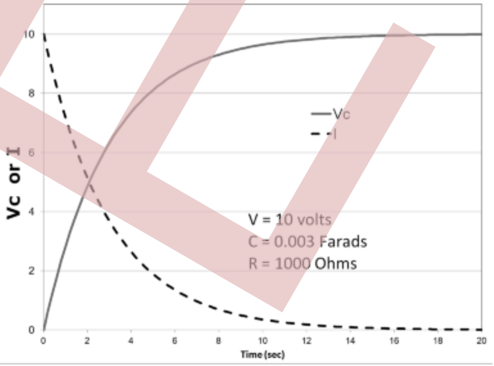
$V = q / C$ where q is the charge in coulombs and C is the capacitance in Farads

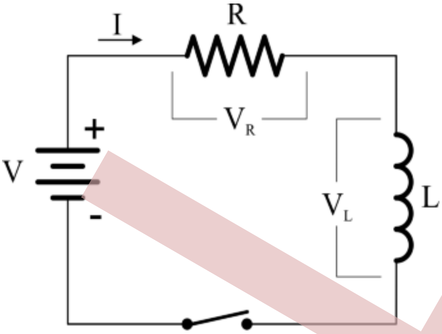
This leads to some very interesting circuit behavior. Consider the following circuit. Without the capacitor, current would flow immediately when the switch was closed with a value:



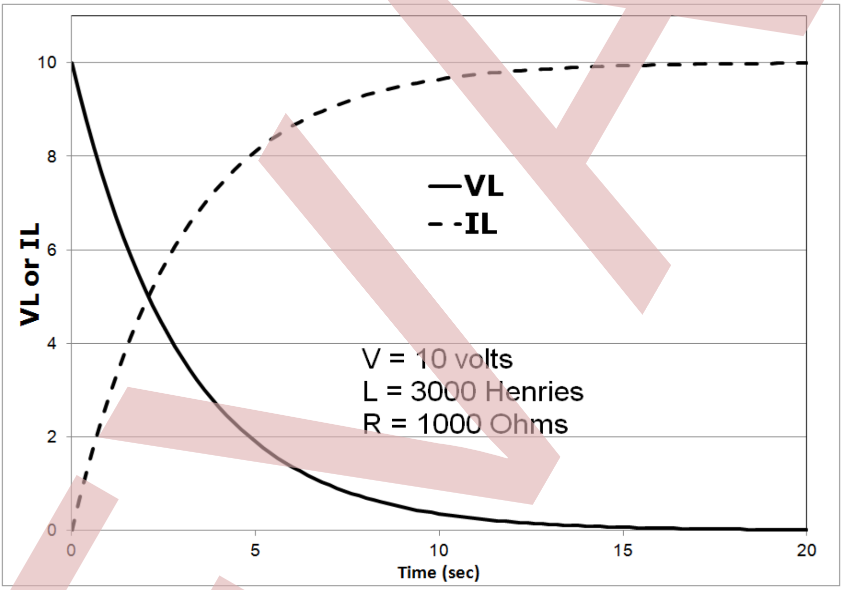
$I = V / R$

With the capacitor in the circuit, the initial current flowing is the same, because the charge on the capacitor is zero so the voltage across the capacitor is zero. As time progresses, current flows into the capacitor. This causes the voltage across the capacitor to increase. The voltage across the resistor then decreases (KVL) so the current goes down. As a result we get the following time dependent voltage and current waveforms.





The inductor is the other basic linear circuit component. While capacitors store charge, inductors store energy in a magnetic field. When you apply a voltage to a capacitor it starts at zero and increases as the charge builds in the capacitor. When you apply a voltage to an inductor, the voltage immediately goes to full value, then as the magnetic field builds the voltage across the inductor drops to zero. Note that this plot is identical to the plot for the capacitor except the voltage and current are swapped. The time dependent behavior of these two circuit components becomes very important as we move on to discussion of Alternating Current (AC) circuits.



Facts:

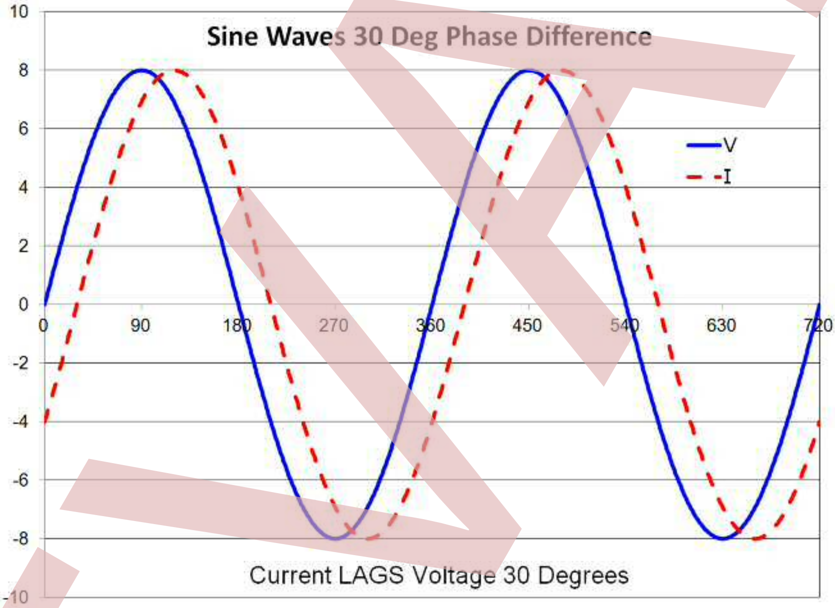
Ohm	Unit of measure for resistance. A resistor impedes the flow of current.
Farad	Unit of measure for capacitance. A capacitor stores energy in the form of electric charge. A capacitor reduces the rate of change of an applied voltage.
Henry	Unit of measure for inductance. An inductor stores energy in a magnetic field. An inductor reduces the rate of change of a current flowing through it.

Alternating Current Circuits

Almost all electric power systems use alternating current (AC). AC is easier to produce and distribute because transformers allow easy, efficient conversion between various voltage levels.

Since power is the product of voltage and current, by raising the voltage for distribution, less current is required to distribute the same power. Less current means loss in the conductors is less, so the power distribution system is more efficient.

In DC circuits we only had one parameter to consider, the amplitude of the signal. In sinusoidal AC circuits there are three parameters of importance:



AValue	When we speak of the value of an AC waveform we generally are speaking about its root mean squared (RMS) value. For a sine wave the RMS value of the waveform is related $V_{rms} = V_{peak} / \sqrt{2}$ to its peak amplitude by
Frequency	The number of full sinusoidal oscillations per second of the signal. For power distribution in the US this is 60 cycles per second. Frequency is generally specified in units of Hertz (Hz) or kilohertz (kHz). 1 Hz = 1 cycle per second.
Phase	The fractional part of a cycle through which a waveform has progressed compared to a reference. Phase is generally measured in degrees where there are 360 degrees in a cycle.

AC Circuit Laws

We choose to define the amplitude of an AC signal so the same circuit laws apply for AC circuits as for DC circuits. Specifically, so that

$V = IR$ Ohms's Law

and $P = I^2R = V^2/R$ Joule's Law

For the DC amplitude to equal the AC amplitude we need.

$$\frac{V_{dc}^2}{R} = \int \frac{1}{R} V_0^2 Sin^2(2\pi ft - \theta)dt$$ Ohms's Law

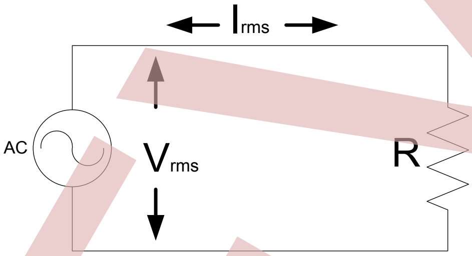
$$\frac{V_{dc}^2}{R} = \frac{V_0^2}{2R}$$ Joule's Law

Which $V_0 = \sqrt{2}V_{DC}$ implies

For a sinusoidal waveform with a **value** of 120 volts, $V_0 = 169.68$.

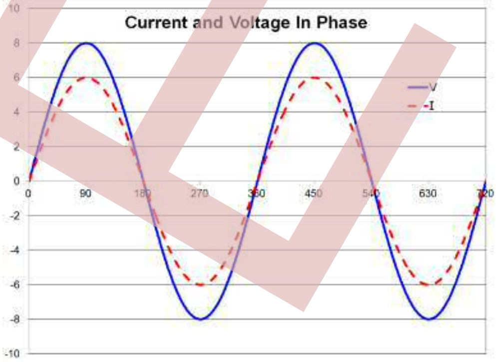
Whether the waveform is a simple sine wave or an extremely complex harmonic one, the RMS (root mean squared) amplitude defined below is the ***value*** we assign to the waveform for the various circuit laws to apply.

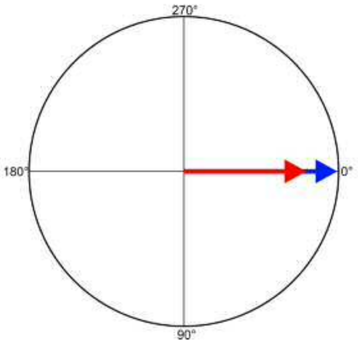
$$V_{rms} = \sqrt{\int [F(2\pi ft)]^2 d}$$ where *F(2πft)* is the waveform



Resistive Load

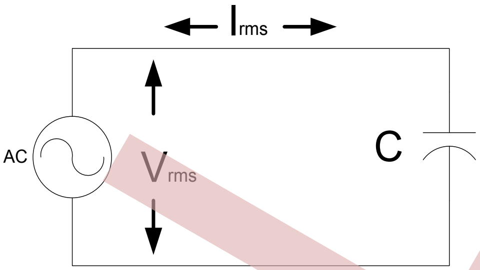
When a sinusoidal voltage is applied to a resistive load the voltage and current are in phase, i.e. the phase angle between them is zero.

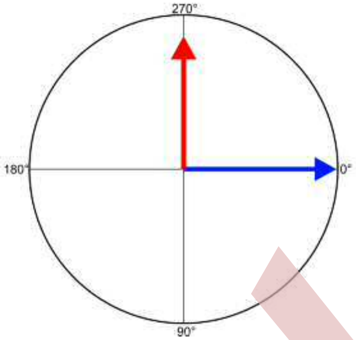


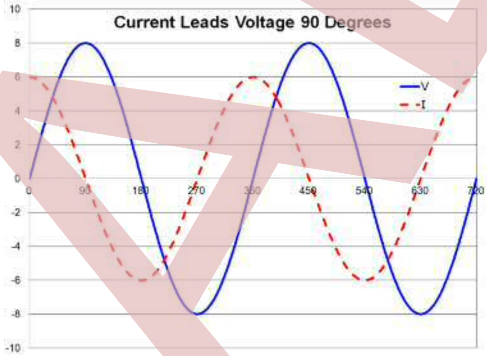


Capacitive Load

When a sinusoidal voltage is applied to a capacitive load the voltage and current are 90 degrees out of phase with the current leading the voltage.

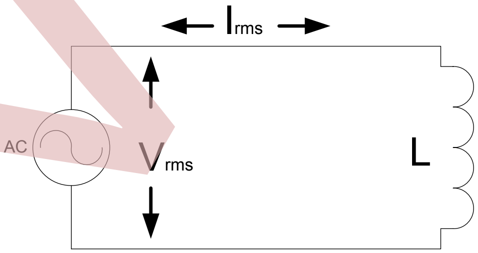


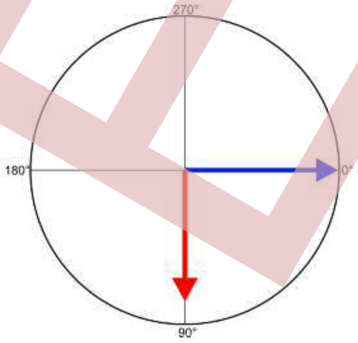


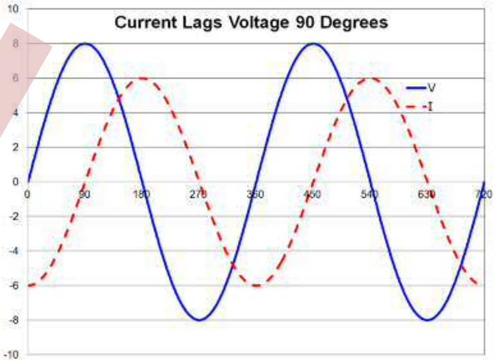


Inductive Load

When a sinusoidal voltage is applied to a inductive load the voltage and current are 90 degrees out of phase with the current lagging the voltage.







Power (Sinusoidal)

Some basic definitions:

Active Power: (1) Instantaneous rate at which work is done.
(2) For metering: The time average of the instantaneous power over one period of the wave. (IEEE 100, C12.1)

If voltage and current are given by:

$$V(t) = \sqrt{2}V_{rms}Sin(\omega_0 t) \quad I(t) = \sqrt{2}I_{rms}Sin(\omega_0 t - \theta)$$

Then the power is given by:

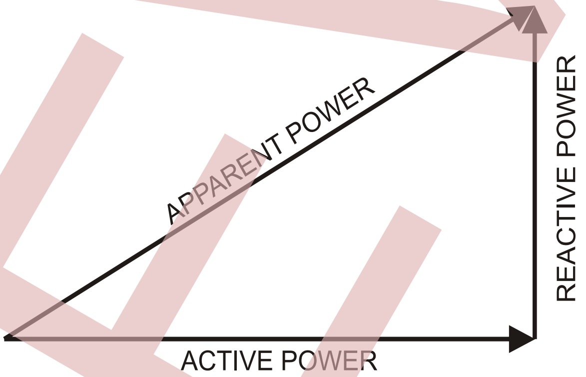
$$P = \frac{1}{kT}\int_0^{kT} V(t)I(t)dt = \frac{1}{kT}\int_0^{kT} \sqrt{2}V_{rms}Sin(\omega_0 t)\sqrt{2}I_{rms}Sin(\omega_0 t - \theta)dt$$

$$P = V_{rms}I_{rms}Cos(\theta)$$

Active power is measured in units of watts (W), kilowatts (kW) or megawatts (MW).

Active Energy: Is the integral of the active power over a period of time. If the period of time is not an exact number of periods then corrections must be applied for the fractional period.

Apparent Power: The product of rms current and rms voltage for any wave form in a two-wire circuit. For **sinusoidal quantities**, apparent power is equal to the square root of the sum of the squares of the active and reactive powers in both two-wire and polyphase circuits. (IEEE 100, C12.1).



The concept of apparent power for sinusoidal waveforms introduces us to the concept of the “Power Triangle”. The Power Triangle is a right triangle where the base is active power, the hypotenuse is apparent power, and the opposite side is designated as reactive power.

The concept of reactive power comes from the term reactance. As we saw earlier both capacitors and inductors shift current waveforms by 90 degrees. Capacitors and inductors are said to have reactance for this reason. This results in a situation where even though there are volts and amps flowing, there is no work performed, hence no active power.

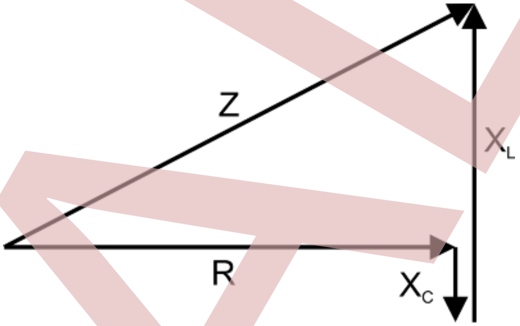
In an AC circuit, the flow of current is limited by any reactance.

R resistance
X_L inductance
X_C capacitance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Z total impedance

where



Apparent power is measured in units of volt-amperes (VA), kilovolt-amperes (kVA), or megavolt-amperes (MVA).

For polyphase systems two distinct approaches are used to compute system active power:

Arithmetic: $$S_{sys} = |S_a| + |S_b| + |S_c|$$

Vectorial: $$\overrightarrow{S_{sys}} = \overrightarrow{S_a} + \overrightarrow{S_b} + \overrightarrow{S_c}$$

The vectorial approach is the most commonly used since it gives the expected answers in many common metering applications.

Reactive Power: (1) For metering: For sinusoidal quantities in a two-wire circuit, reactive power is the product of the voltage, the current, and the sine of the phase of the angle between them. For non-sinusoidal quantities, it is the sum of all harmonic components, each determined as above. (IEEE 100).
(2) For electrical measurements: The square root of the square of the apparent power S minus the square of the active power P. (IEEE 100)

$$Q = \sqrt{S^2 - P^2}$$

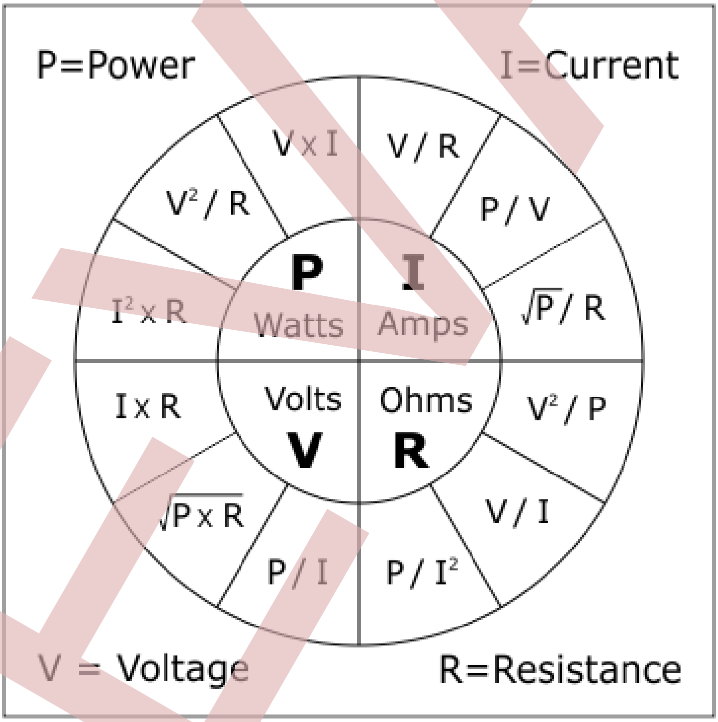
Reactive power is measured in units of volt-amperes reactive (VAR), kilovolt-amperes reactive (kVAR), or megavolt-amperes reactive (MVAR).

Power Factor: (1) For metering: The ratio of the active power to the apparent power. (IEEE 100, C12.1).

$$PF = P / S = P / \sqrt{P^2 + Q^2}$$

Power factor, as it has been discussed so far, is a sinusoidal assumption concept. In the real world of today many loads create highly non-sinusoidal waveforms. When one generalizes the concepts of VA, VAR and power factor to real world situations, things get complicated. Today in the US, there is no legal definition of any of these quantities in non-sinusoidal situations. Later a 21st century approach is presented,. However, we need to keep in mind that today, with no legal definition or standard, different meters may measure different values when waveforms are not sinusoidal.

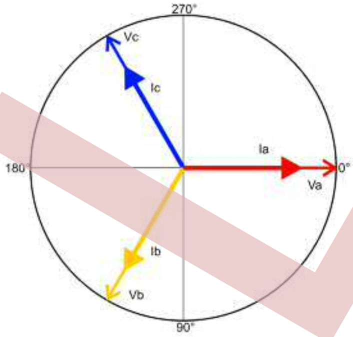
Summary: Basic Power Theory



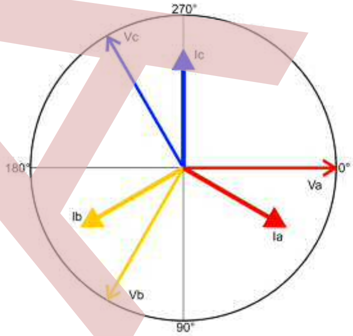
The most fundamental relationships we use in metering are the relationships between voltage, current, resistance and power. The "power circle" gives all of the different relationships.

Summary: Vector Diagrams

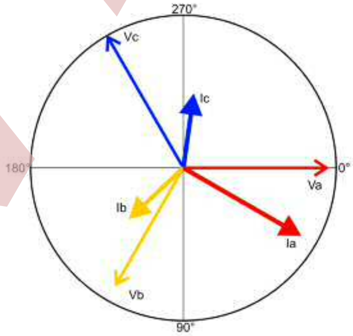
One of the most useful tools for understanding metering installations is the vector diagram. In a vector diagram we represent the voltages and currents by arrows on a polar plot. The relative amplitudes of the polyphase voltages and currents are plotted.



This is the vector diagram of a 4-wire wye service at a power factor of 1.00 with balanced voltages and currents. The voltage and current vectors of each phase are aligned. They are separated by 120 degrees. The currents all have equal amplitudes.

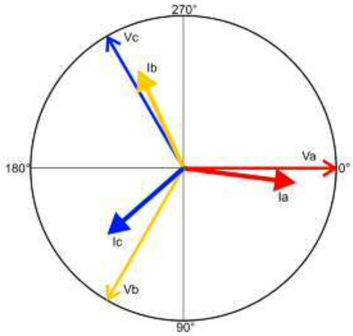


Here the voltages and currents remain in balance but the power factor is 0.866 lagging. The current vectors are shown at 30, 210 and 270 degrees.



	Phase A	Phase B	Phase C
Voltage	118	116	120
V Phase	0°	120°	239°
Current	165	67	73
C Phase	29°	132°	276°
PF	0.874	0.978	0.798

Here the voltages and currents are unbalanced with different power factors for each phase.



The vector diagram is also an excellent tool for finding wiring errors in a system. The diagram at the right shows a 4-wire wye with the phase B and C currents reversed at the point of measurement. This might be a wiring error or a misplacement of the test leads.

Power Measurement in the Digital World

The previous equations apply only to systems with sinusoidal waveforms. In the modern world of complex non-linear loads, voltage and current are seldom pure sinusoids, and are often very highly distorted. Because loads can now be electronically switched, they can also vary dramatically over very short periods of time. In the world of mechanical power meters today's load waveforms presented unsolvable measurement problems.

Today all modern digital meters measure voltage and current waveforms by digitizing them using many samples per cycle. This has changed the approaches that we can use for measuring power quantities. There are two fundamentally different approaches that can be used: 1) computation in the time domain and 2) computation in the frequency (Fourier) domain. Provided that the waveforms are repetitive over the number of cycles in the FFT, both approaches give identical answers for active power (P) and apparent power (S). There is no direct computation for reactive power in the time domain.

Measurement of active power in the time domain is very straight forward. We need simply sum up the product of instantaneous voltage and current over the period of time we desire (provided that the time period is an integral number of cycles).

$$P = \frac{1}{N}\sum_{i=0}^{nN} V_i I_i$$

where N is the number of measurements per cycle

Apparent power is also quite easy to measure in the digital world. We can directly calculate the RMS voltage and current.

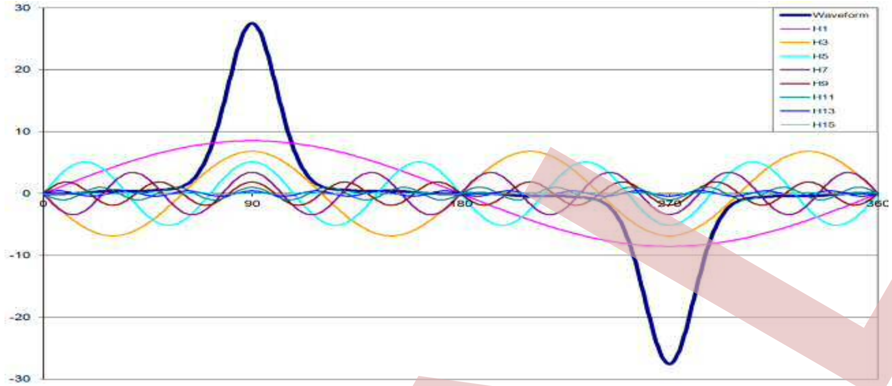
$$S = V_{rms} I_{rms} = \sqrt{\tfrac{1}{N}\sum_{i=0}^{i=N-1} V_i^2 \bullet \tfrac{1}{N}\sum_{i=0}^{i=N-1} I_i^2}$$

Reactive power cannot be computed directly from the data in the time domain. For that reason, in the time domain we derive Q from the other two directly measured quantities as:

$$Q = \sqrt{S^2 - P^2}$$

Another modern computational approach to power quantities is the use of Fourier transform methods. These methods start with the exact same voltage and current measurements as in the time domain but use a technique called an FFT to decompose complex waveforms into a sum of harmonic sine waves which can then be dealt with like the simple sinusoidal case.

Many electronic devices present current loads similar to that shown below. This very non-sinusoidal waveform can be represented by a sum of sine waves of multiples of 60Hz. When a waveform is analyzed using an FFT we get a set of sine/cosine waves at multiples of the fundamental frequency (60 Hz).



Waveforms can be expressed as either sums of sine with phase or sums of sines and cosines.

$$V(t) = \sqrt{2}\sum_{n=1}^{\infty}\left(V_n Sin(n\omega_0 t - \alpha_n)\right) = \sum_{n=1}^{\infty}\left(a_n Cos(n\omega_0 t) + b_n Sin(n\omega_0 t)\right)$$

$$I(t) = \sqrt{2}\sum_{n=1}^{\infty}\left(I_n Sin(n\omega_0 t - \beta_n)\right) = \sum_{n=1}^{\infty}\left(c_n Cos(n\omega_0 t) + d_n Sin(n\omega_0 t)\right)$$

Using this approach we can compute the various power quantities as:

$$P = \sum_{n=1}^{\infty} V_n I_n Cos(\theta_n) = \sum_{n}\left[a_n c_n + b_n d_n\right] \quad \text{where} \quad \theta_n = \alpha_n - \beta_n$$

$$S = \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\sum_{n=1}^{\infty}(c_n^2 + d_n^2)$$

$$Q = \sum_{n=1}^{\infty} V_n I_n Sin(\theta_n) = \sum_{n}\left[a_n c_n - b_n d_n\right]$$

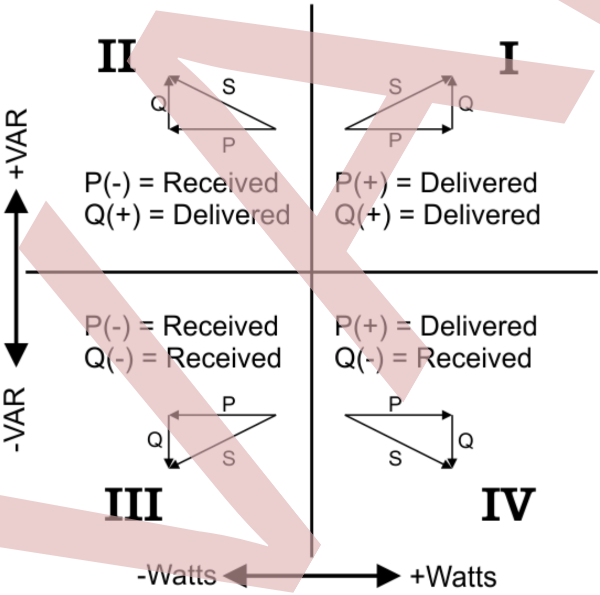
The historical approach of calculating active power and reactive power was driven by the technology available in the early days of electric power. These approaches ignored harmonics and rapidly changing non-harmonic situations. Today these situations are the norm. Today even though technology is available to accurately compute all power quantities directly from the sampled data, many meters still emulate the old approaches so their answers will be consistent with the old results.

With today's technology the approaches defined above can easily be implemented so as to provide consistent and accurate measurement of all power quantities. Though active power is generally how energy is currently billed, apparent power better measures the real cost of providing the service to the customer. Since computing apparent power is easy to do with modern technology, we should be looking to switching to it as the measure of energy for the future.

Bidirectional Metering

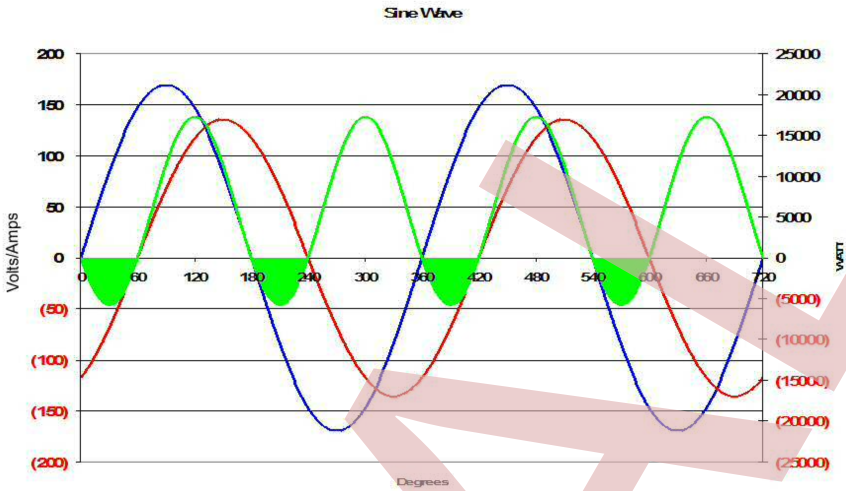
Bidirectional metering has become very important in this day of solar arrays, wind power and other types of customer cogeneration. When energy can flow in either direction between the customer and utility, it is important to understand exactly how this situation should be metered.

Bidirectional power flow is described in terms of both active power (watts) and reactive power (VARs). Measurement of bidirectional flows are often referred to as "four quadrant" power analysis. In quadrants I and IV power is delivered to the customer. In quadrants II and III power is supplied by the customer to the utility, he is cogenerating.



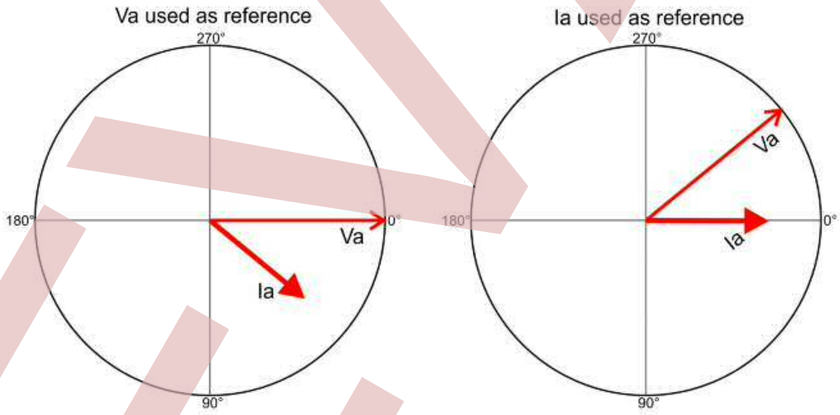
When bidirectional power flows exist the application can be metered in two basically different approaches. In the "net metering" approach power in both directions is measured but only the net power reported. Alternatively, power and VAR are recorded in multiple registers (or using multiple meters with detents) so the total delivered and received are independently recorded and reported. This approach allows power generated by the customer to be billed at a different rate than power supplied by the utility.

On an instantaneous basis power often flows in both directions. However, for there to be true received power there must be net power flow on a full cycle basis. In the plot opposite the solid shaded area is power flowing instantaneously from the customer; however when a full cycle is measured the flow is from the utility. No **received** power would be measured.



Some confusion often arises when looking at vector diagrams to determine the quadrant of energy flow.

In the IEEE definition of power vectors *Ia* is specified as the reference vector. However, most measurement devices use *Va* as the reference along the positive X axis. This is done because in live measurements *Va* has a large, stable value that is easily measured; while *Ia* may be small and vary quite significantly in time. Hence a display with *Va* as the reference appears stable, while a display with *Ia* as the reference jitters and looks quite different depending on the power factor. When the vector diagram is plotted with Ia as the reference then Va points to the appropriate quadrant for the direction of power and VAR.



When Ia is used as reference then Va points to the correct quadrant for delivered/received power.